



## Numbers Early Learning Goal

- Solve problems, including doubling and halving.

### Exceeding judgement

- Solve practical problems that involve combining groups of 2, 5 or 10.

## Models and Strategies

Where possible, concepts should be taught in the context of real life.

**Counting in repeated groups of the same size** using real-life contexts and practical apparatus.



2... 4... 6 socks

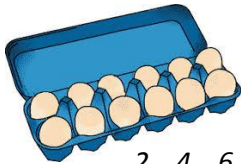
- Use pictorial representations alongside real objects (e.g. pairs of socks on a line or wellies on a rack).
- Sing, count and chant in twos, fives and tens (with and without objects).

### Use of the 100 square to identify patterns.

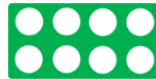
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

### Introduce simple arrays using real objects.

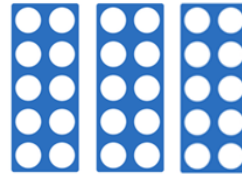
E.g. real eggs in a box, Numicon pieces, Numicon peg board, sorting trays.



2... 4... 6... 8... 10... 12 eggs



Two, four, six, eight



Ten, twenty, thirty

- Start to combine groups by counting on from a set.
- E.g. *Let's put 4 in our heads and carry on counting in twos. 4, 6, 8, 10.*



Four, six, eight, ten

### Solve practical problems that involve counting in twos, fives and tens.

- E.g.  *How many wheels are there altogether?*

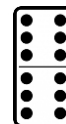
### Understand doubling as adding the same number.



Double 2 is 4



Double 3 is 6



- Begin to recall doubles and halves using songs and games.
- E.g. songs with actions - 'Mr Double -Trouble'; doubling machine games; finding doubles in dominoes.

### Recording.

- Record calculations in pictures - Numicon pieces can be drawn around.

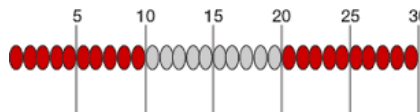


## Objectives

- Count in multiples of twos, fives and tens.
- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Double numbers and quantities.
- Make connections between arrays, number patterns, and counting in twos, fives and tens.

## Models and Strategies

### Counting in twos, fives and tens.



### Understand multiplication in terms of counting groups of the same size (repeated addition).

- Use of concrete apparatus (e.g. washing line and other practical resources for counting, Numicon, bundles of straws, bead strings) and pictorial representations.

E.g. *There are 5 pairs of socks. How many socks are there altogether?*

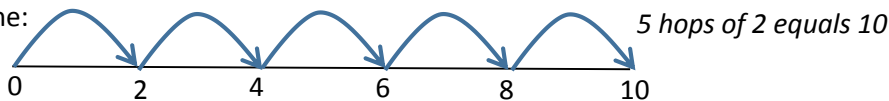


$$2 + 2 + 2 + 2 + 2 = 10$$

or

$$5 \text{ lots of } 2 \text{ equals } 10$$

- Record as jumps on a number line:



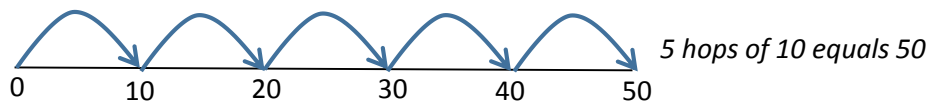
Sam has five 10p coins. How much does Sam have in total?



$$10 + 10 + 10 + 10 + 10 = 50p$$

or

$$5 \text{ lots of } 10p \text{ equals } 50p$$



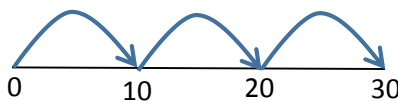
### Using arrays and Numicon to understand multiplication.



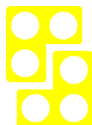
$$5 + 5 + 5 = 5 \text{ lots of } 3$$



$$10 + 10 + 10 = 3 \text{ lots of } 10$$



### Doubling numbers to 10 progressing to doubling numbers to 30.



$$\text{Double } 3 = 3 + 3 = 2 \text{ lots of } 3$$



$$\text{Double } 6 = 6 + 6 = 2 \text{ lots of } 6$$

### Language of multiplication.

- Use concrete apparatus to develop the vocabulary relating to 'times'.

E.g. *Pick up five, 4 times.*





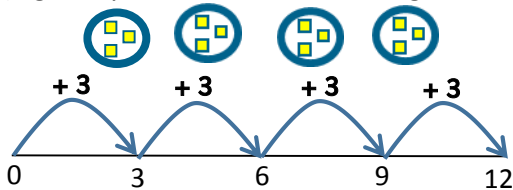
## Objectives and additional guidance

- Count in steps of 2, 3, and 5 from 0, and in tens from any number, forward or backward.
- Recall and use facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Show that multiplication of two numbers can be done in any order (commutative).
- Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication ( $\times$ ) and equals ( $=$ ) signs.
- Solve problems involving multiplication, using materials, arrays, repeated addition, mental methods, and multiplication facts, including problems in contexts.
- Use a variety of language to describe multiplication.
- Connect 10 times multiplication table to place value and 5 times multiplication table to divisions on a clock face.
- Count in multiples of 3 to support later understanding of a third (from *Number and Place Value*).
- Begin to use other multiplication tables and recall facts to perform written and mental calculations.
- Use commutativity and inverse relations to develop multiplicative reasoning (e.g.  $4 \times 5 = 20$  and  $20 \div 5 = 4$ ).

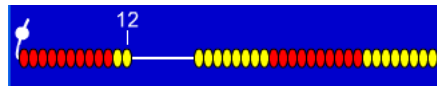
## Models, Strategies and Methods

### Repeated addition, arrays and number lines.

Continue to develop understanding of multiplication as repeated addition using a range of images (e.g. arrays, number lines, counting stick, bead bars/strings).

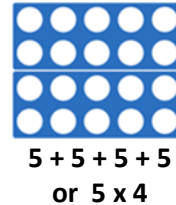
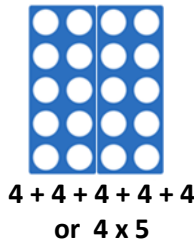
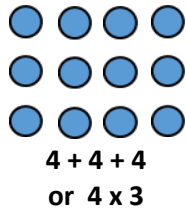
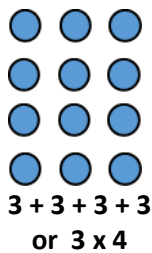


4 lots of 3 makes 12  
 3 multiplied by 4 equals 12



Begin to understand that in this example, 4 is the multiplier, so it is written as  $3 \times 4$  (four groups of 3 or three, four times). This will help their understanding of the scaling model of multiplication in Year 3.

### Use arrays to understand that multiplication can be done in any order (commutative).



### Express multiplication as a number sentence using the 'x' sign.

Use understanding of the **inverse operation** and practical apparatus, to solve missing number problems.

$7 \times 2 = \square$        $\square = 2 \times 7$        $7 \times \square = 14$        $14 = \square \times 7$   
 $\square \times 2 = 14$        $14 = 2 \times \square$        $\square \times \bigcirc = 14$        $14 = \square \times \bigcirc$

### Develop fluency in mental recall of the 2, 5 and 10 times tables.

Use a variety of multi-sensory approaches to aid memorisation of times table facts: verbal repetition through songs (e.g. musicalmaths.com) and actions; use of images to support understanding; playing games (e.g. 'Fizz, buzz'); investigate patterns in the 100 square.

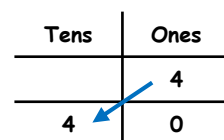
Strike out the multiples of 2 and circle the multiples of 5.  
 What do you notice?

1	<del>2</del>	3	<del>4</del>	5	<del>6</del>	7	<del>8</del>	9	<del>10</del>
11	<del>12</del>	13	<del>14</del>	15	<del>16</del>	17	<del>18</del>	19	<del>20</del>
21	<del>22</del>	23	<del>24</del>	25	<del>26</del>	27	<del>28</del>	29	<del>30</del>

Recall doubles to 10 and understand doubling as multiplying by 2.

### Begin to understand the effect of multiplying by 10 on place value.

E.g. In  $4 \times 10$  the digit moves one place to the left.  
 The zero is the place value holder. Do **NOT** refer to as 'adding a zero'.





## Objectives and additional guidance

Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.

Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for 2-digit numbers times 1-digit numbers, using mental methods and progressing to the formal written method of short multiplication.

Solve simple problems, including missing number problems, involving multiplication and division, including positive integer scaling problems (e.g. 4 times as high, 8 times as long) and correspondence problems in which  $n$  objects are connected to  $m$  objects (e.g. 3 hats and 4 coats - how many different outfits?).

Continue to practise mental recall of multiplication tables to develop fluency. Through doubling, connect the 2, 4 and 8 multiplication tables.

Develop efficient mental methods; for example, using commutativity (e.g.  $4 \times 5 = 5 \times 4$ ) and associativity [e.g.  $2 \times (3 \times 4) = (2 \times 3) \times 4$ ], and using multiplication/division facts (e.g.  $3 \times 2 = 6$ ,  $6 \div 3 = 2$  and  $2 = 6 \div 3$ ) to derive related facts ( $30 \times 2 = 60$ ,  $60 \div 3 = 20$  and  $20 = 60 \div 3$ ).

## Models, Strategies and Methods

### Develop fluency in mental recall of the 3, 4 and 8 times tables.

Use a variety of multi-sensory approaches to aid memorisation of times table facts (see Year 2).

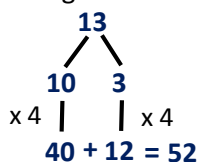


Multiply by 4 and 8 respectively, by doubling, doubling again ( $\times 4$ ) and doubling again ( $\times 8$ ).

### Mental strategies.

Multiply two-digit numbers by one-digit numbers by **partitioning**.

E.g.  $13 \times 4 = 10 \times 4 + 3 \times 4$   
 $= 40 + 12$   
 $= 52$



### Multiplying one-digit and two-digit numbers by 10 and 100.

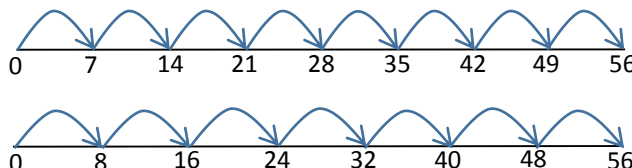
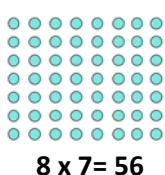
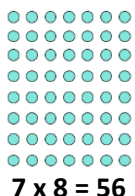
E.g.  $13 \times 100 = 1300$ . The digits move two places to the left.

The zeros are the place value holders. Do **NOT** refer to 'adding zeros'.



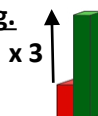
### Continue to use arrays to understand the commutative nature of multiplication.

E.g.



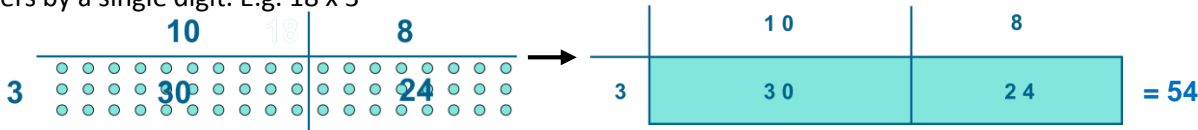
### Begin to develop an understanding of multiplication as scaling.

E.g. The green rod is **three times taller** than the red rod.



### Begin to develop a written method.

Some children will use arrays to begin to develop a written method (Grid Method) to multiply two-digit numbers by a single digit. E.g.  $18 \times 3$



### Solve missing number problems.

Continue with a range of equations as in Year 2 but with appropriate numbers.

E.g.  $7 \times 8 = \square$        $\square \times 8 = 56$        $12 \times \square = 1200$        $5 \times 6 = 2 \times \square$

**Objectives and additional guidance**

Recall multiplication facts for all times tables up to  $12 \times 12$ .

Use place value, known and derived facts to multiply mentally, including multiplying by 0 and 1; multiplying together 3 numbers. Extend to deriving facts for 3-digit numbers (e.g.  $600 \div 3 = 200$  can be derived from  $2 \times 3 = 6$ ).

Recognise and use factor pairs and commutativity in mental calculations.

Multiply 2-digit and 3-digit numbers by a 1-digit number using formal written layout (short multiplication).

Solve problems involving multiplying and adding, including use of distributive law to multiply 2-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as  $n$  objects are connected to  $m$  objects (e.g. number of choices of a meal on a menu).

Write statements about equality of expressions; for example, use of distributive law (e.g.  $39 \times 7 = 30 \times 7 + 9 \times 7$ ) and associative law (e.g.  $(2 \times 3) \times 4 = 2 \times (3 \times 4)$ ).

Combine knowledge of number facts and rules of arithmetic to solve mental and written calculations (e.g.  $2 \times 6 \times 5 = 10 \times 6 = 60$ ).

Solve 2-step problems in context, choosing appropriate operation, working with increasingly harder numbers.

**Models, Strategies and Methods****Develop fluency in mental recall of multiplication facts up to  $12 \times 12$ .**

Use a variety of multi-sensory approaches to aid memorisation of multiplication facts (see Year 2).

**Mental strategies.**

Use known multiplication facts to derive associated facts.

E.g.  $8 \times 7 = 56$ , so  $80 \times 7 = 560$  and  $80 \times 70 = 5600$ .

Multiply three numbers together and use the associative law.

E.g.  $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$

Multiply two-digit numbers by one-digit numbers by **partitioning**. E.g. deriving doubles.

$$\begin{aligned} 37 \times 2 &= 30 \times 2 + 7 \times 2 \\ &= 60 + 14 \\ &= 74 \end{aligned}$$

Multiply one-digit and two-digit numbers by 10 and 100.

E.g.  $34 \times 100 = 3400$ . The zeros are the place value holders.

Do **NOT** refer to as 'adding zeros'.

1000s	100s	10s	1s
		3	4
3	4	0	0

Blue arrows indicate the shift of digits: 3 from 10s to 1000s, 4 from 1s to 100s.

**Understand multiplication as scaling.**

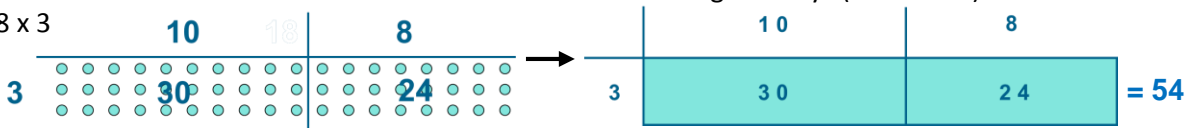
Solve practical problems where pupils need to scale up. Relate to known number facts.

E.g. *My sunflower is 25cm tall. Yours is 6 times taller. How tall is your sunflower?*

**Using a written method (Grid Method).**

Revise and deepen pupils' understanding of the Grid Method to multiply two-digit numbers by 1-digit numbers. Ensure that this is still linked back to their understanding of arrays (see Year 3).

E.g.  $18 \times 3$



Extend use of Grid Method to multiply three-digit numbers by one-digit numbers.

E.g.  $235 \times 6$

$\times$	200	30	5	
6	1200	180	30	
				1200
				+ 180
				30
				<u>1410</u>
				1

**Solve missing number problems.**

Continue with a range of equations as in Year 3, but also include equations with missing digits.

E.g.  $2 \square \times 5 = 115$



## Objectives and additional guidance

- Recall and use multiplication facts for all times tables up to  $12 \times 12$ .
- Identify multiples and factors; find all factor pairs of a number and common factors of 2 numbers.
- Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Establish whether a number up to 100 is prime and recall prime numbers up to 19.
- Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method.
- Multiply numbers mentally, drawing upon known facts.
- Multiply whole numbers and those involving decimals by 10, 100 and 1,000.
- Recognise and use square numbers and cube numbers, and the notation for squared ( $^2$ ) and cubed ( $^3$ )
- Solve problems involving multiplication, including: (a) using knowledge of factors and multiples, squares and cubes; (b) scaling by simple fractions and problems involving simple rates.
- Use multiplication to convert between units of measure.
- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams.
- Connect multiplication by a fraction to using fractions as operators (fractions of).

## Models, Strategies and Methods

### Mental strategies.

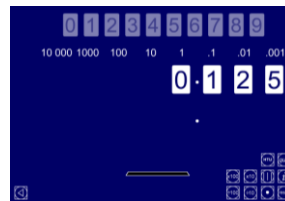
Ensure fluent recall of multiplication facts up to  $12 \times 12$ , and use confidently to derive associated facts. E.g.  $8 \times 7 = 56$ , so  $0.8 \times 7 = 5.6$  and  $0.8 \times 0.7 = 0.56$ .

Solve problems such as  $15 \times 9$  using an efficient mental strategy.

E.g. Partitioning:  $15 \times 9 = (10 \times 9) + (5 \times 9) = 90 + 45 = 135$ ; multiply 15 by 10, then subtract 15.

Multiply whole numbers and decimal numbers by 10, 100 and 1000. Use place value charts to support understanding (e.g. Gattengo chart; *Moving Digits* ITP).

ten thousands	10 000	20 000	30 000	40 000	50 000	60 000	70 000	80 000	90 000
thousands	1000	2000	3000	4000	5000	6000	7000	8000	9000
hundreds	100	200	300	400	500	600	700	800	900
tens	10	20	30	40	50	60	70	80	90
units	1	2	3	4	5	6	7	8	9
tenths	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
hundredths	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
thousandths	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009



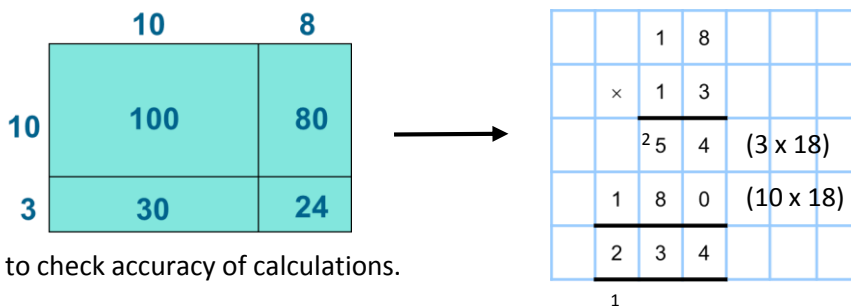
### Understand multiplication as scaling.

Solve practical scaling problems (e.g. scaling up the dimensions of a model or building from a photograph).

### Progress to the formal written methods of multiplication.

Extend use of the Grid Method to multiply three and four-digit numbers by one- and two-digit numbers. Use the Grid Method to support an understanding of the standard algorithm (**short** and **long** multiplication).

E.g.  $18 \times 13$



Use estimation to check accuracy of calculations.

### Solve a range of missing number problems.

E.g.  $53 \div \square = 0.053$

### Multiply proper fractions and mixed numbers by whole numbers.

Connect finding the fraction of a number (e.g.  $\frac{2}{5}$  of 20) to multiplication by a fraction ( $20 \times \frac{2}{5}$ ).

Use Cuisenaire rods and bar model to support multiplication of fractions.

E.g.  $1\frac{1}{3} \times 4 =$





## Objectives and additional guidance

- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.
- Multiply whole numbers and decimals by 10, 100 and 1,000.
- Use knowledge of the order of operations to carry out calculations involving the 4 operations.
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.
- Solve problems involving the four operations.
- Identify common factors, common multiples and prime numbers.
- Multiply simple pairs of proper fractions, writing the answer in its simplest form.

## Models, Strategies and Methods

### Mental strategies.

- Ensure fluent recall of multiplication facts up to  $12 \times 12$ , and derive associated facts (e.g.  $0.8 \times 0.7 = 0.56$ ).
- Derive associated facts from known multiplication facts.  
E.g. If  $126 \times 42 = 5292$ , use the this fact to work out:  $12.6 \times 42$ ;  $12.6 \times 4.2$ ;  $12.6 \times 4200$ .
- Solve problems such as  $15 \times 9$  using an efficient mental strategy (see Year 5).
- Multiply whole numbers and decimal numbers by 10, 100 and 1000 (see Year 5).

### Identify common multiples of given numbers.

E.g. Find common multiples of 40 and 90.

### Use BIDMAS to solve problems that use knowledge of the order of operations.

E.g. Understand how to solve  $7 + 4 \times 3$  and  $(7 + 4) \times 3$ .

### Understand multiplication as scaling.

Solve practical scaling problems (e.g. scaling up the dimensions of a model or building from a photograph).

### Formal written methods to multiply up to 4 digit numbers by 2 digit whole numbers.

Develop fluency and deepen understanding of the **compact** written methods of short and long multiplication. Use estimation to check answers to calculations.

E.g.  $567 \times 36$

Estimate =  $600 \times 40 = 24,000$

		5	6	7
×			3	6
<hr/>				
	3	44	40	2
1	27	20	1	0
<hr/>				
12	0	4	1	2

Extend to multiplying decimals (e.g.  $6.42 \times 32$ ).

### Solve a range of missing number and missing digit problems.

E.g.  $24 \times \square = 168$

Find the missing digits:

	2		
×		8	
4	1	9	2

### Multiply simple pairs of proper fractions.

E.g.  $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

Use diagrams to support understanding.

